SIR VEN
(GURU ADIWIRA KEBANGSAAN 2019)

SPM 2023

SOALAN RAMALAN

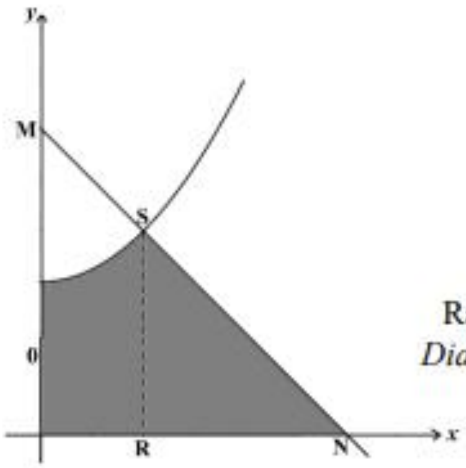
MATEMATIK TAMBAHAN

KERTAS 2

PENGAMIRAN

INTEGRATION

11



Rajah 6
Diagram 6

Dalam rajah 6, garis lurus MN ialah normal kepada lengkung $y = \frac{x^2}{3} + 4$ pada titik $(3,5)$. Koordinat titik N ialah $(k,0)$. Garis lurus RS adalah selari dengan paksi- y

In Diagram 6, the straight line MN is normal to the curve $y = \frac{x^2}{3} + 4$ *at point* $(3,5)$. *The coordinates of point N are* $(k,0)$ *.The straight line* RS *is parallel to the* y *-axis*

Cari
Find

(a) nilai k , [3 *markah*]
the value of k , [3 *marks*]

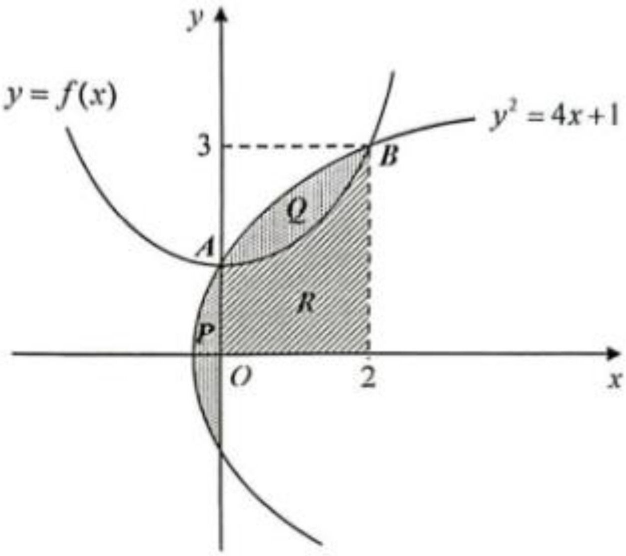
(b) luas, dalam unit^2 , kawasan berlorek, [4 *markah*]
the area, in unit^2, *of the shaded region,* [4 *marks*]

(c) isi padu yang dijanakan dalam sebutan π, apabila rantau yang dibatasi oleh lengkung, garis lurus RS, paksi- x dan paksi- y diputarkan $360°$ pada paksi- x .
volume generated in terms of π *, when the region bounded by the curve, straight line RS, the* x *-axis and the* y *-axis is revolved* $360°$ *about the* x *-axis.*

[3 *markah*]
[3 *marks*]

10 Rajah 3 menunjukkan dua lengkung $y = f(x)$ dan $y^2 = 4x + 1$. Lengkung-lengkung tersebut bersilang pada titik A dan titik B.

Diagram 3 shows two curves $y = f(x)$ and $y^2 = 4x + 1$. The curves intersect at point A and point B.



Rajah 3
Diagram 3

(*a*) Nyatakan koordinat A.
State the coordinates of A.

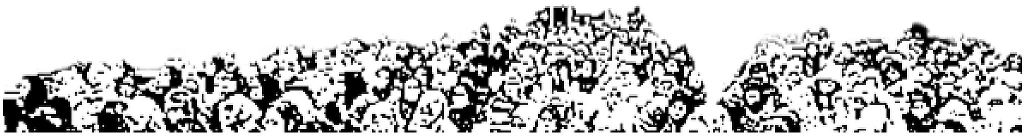
[1 *markah*]
[1 *mark*]

(*b*) Diberi bahawa $\int_0^2 f(x)dx = m$. Cari luas kawasan berlorek P dan Q dalam sebutan m.
It is given that $\int_0^2 f(x)dx = m$. Find the area of the shaded region P and Q in terms of m.

[5 *markah*]
[5 *marks*]

(*c*) Kira isipadu janaan, dalam sebutan π, apabila rantau R dikisarkan 360° pada paksi-x jika luas keratan rentasnya ialah $\frac{1}{4}\pi(x^4 + 4x^2 + 4)$.
Find the volume generated, in terms of π, when the region R is revolved 360° about the x-axis if the area of its cross-section is $\frac{1}{4}\pi(x^4 + 4x^2 + 4)$.

[4 *markah*]
[4 *marks*]



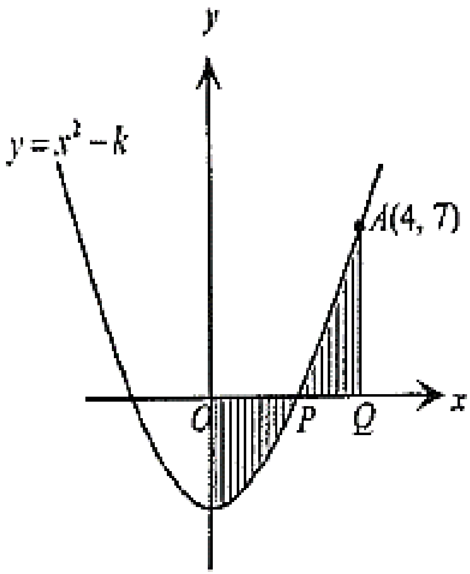
(*a*) Diberi $\frac{d}{dx}\left(3x^2 - x\right) = f(x)$, cari nilai $\int_0^1 f(x)\,dx$.

Given $\frac{d}{dx}\left(3x^2 - x\right) = f(x)$, *find the value of* $\int_0^1 f(x)\,dx$.

(*b*) Fungsi kecerunan suatu lengkung adalah $3x - h$. Tangen kepada lengkung itu pada titik $(5, 7.5)$ memotong paksi-x pada $x = 3.75$.
Cari persamaan lengkung itu. [3 *markah*]

The gradient function of a curve is $3x - h$. *Tangent to the curve at point* $(5, 7.5)$ *cuts the* x*-axis at* $x = 3.75$.
Find the equation of the curve. [3 *marks*]

Rajah 9 menunjukkan sebahagian lengkung $y = x^2 - k$ yang memotong paksi-x di P. Garis lurus AQ selari dengan paksi-y.
Diagram 9 shows the part of the curve $y = x^2 - k$ intersects the x-axis at P. The straight line AQ is parallel to the y-axis.



Rajah 9 / *Diagram 9*

Hitung
Calculate

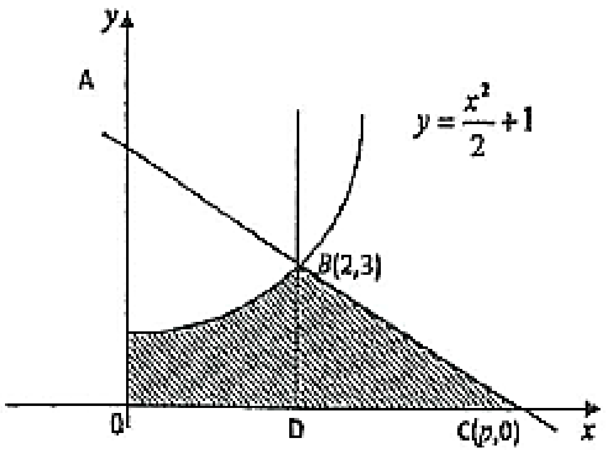
(a) nilai k dan koordinat P, [3 markah
the value of k and the coordinates of P, [3 mark

(b) luas kawasan berlorek, [4 markah
the area of the shaded region, [4 mark

(c) isi padu kisaran, dalam sebutan π, apabila rantau yang dibatasi oleh lengkung dan paksi-diputarkan melalui 180° pada paksi-y. [3 markah
the volume generated, in terms of π, when the region bounded by the curve and x-axis is revolved through 180° about the y-axis. [3 mark

Rajah 9 menunjukkan garis lurus AC yang merupakan garis normal kepada lengkung $y=\frac{x^2}{2}+1$, pada $B(2,3)$. Garis lurus BD adalah selari dengan paksi-y.

Diagram 9 shows a straight line AC which is normal to the curve $y=\frac{x^2}{2}+1$, *at* $B(2,3)$. *The straight line BD is parallel to the y-axis.*



Rajah 9 / *Diagram 9*

Cari
Find

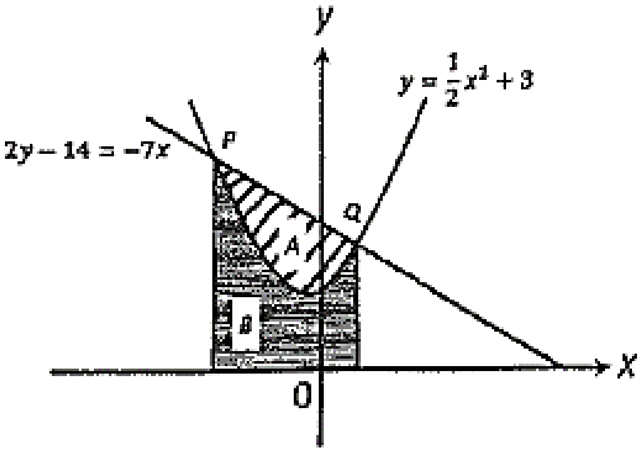
(a) nilai p, [3 *markah*]
the value of p, [3 *marks*]

(b) luas kawasan yang berlorek, [4 *markah*]
the area of the shaded region, [4 *marks*]

(c) isipadu janaan dalam sebutan π, apabila rantau itu dibatasi oleh lengkung tersebut, paksi-y, dan garis lurus $y = 3$ dikisarkan melalui 360° pada paksi-y. [4 *markah*]
the volume generated in terms of π, *when the region bounded by the curve, the y-axis and the straight line* $y = 3$ *is revolved through 360° about the y-axis.* [4 *marks*]

2 Rajah 2 di bawah menunjukkan suatu lengkung $y = \frac{1}{2}x^2 + 3$ yang bersilang dengan garis lur $2y - 14 = -7x$ pada titik P dan titik Q.

Diagram 2 shows the curve $y = \frac{1}{2}x^2 + 3$ *intersects the straight line* $2y - 14 = -7x$ *at the poin* *P and Q.*



Rajah 2 / *Diagram 2*

Cari
Find

(a) koordinat bagi titik P dan titik Q.
the coordinates of point P and point Q.

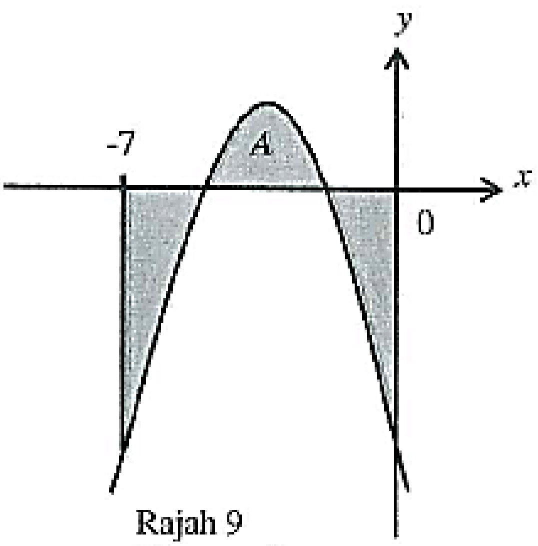
[3 mark
[3 *ma*

(b) luas rantau B, dalam unit^2.
the area of region B, in unit^2.

[4 mark
[4 *ma*

Rajah 9 menunjukkan suatu lengkung $y = f(x)$ di mana $f(x) = -x^2 - 7x - 10$.

Diagram 9 shows a curve $y = f(x)$ *where* $f(x) = -x^2 - 7x - 10$.



Rajah 9
Diagram 9

Hitung
Calculate

(a) jumlah luas kawasan berlorek, [6 *markah*]
the total area of the shaded regions, [6 *marks*]

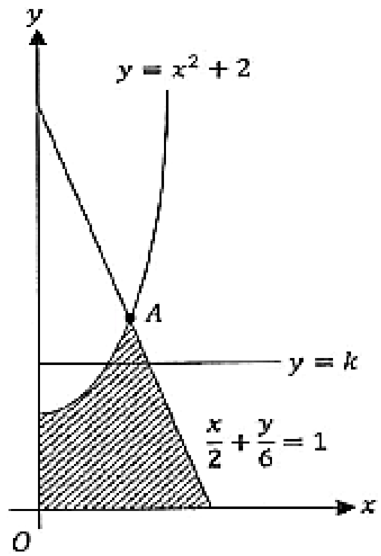
(b) isipadu kisaran, dalam sebutan π, apabila rantau berlorek A dikisarkan melalui 360° pada paksi-x.
the volume of revolution, in terms of π, *when the shaded region A is revolved through* 360° *about the x-axis.* [4 *markah*]
[4 *marks*]

Rajah 2 menunjukkan lengkung $y = x^2 + 2$ yang bersilang dengan garis lurus

$\frac{x}{2} + \frac{y}{6} = 1$ pada titik A.

Diagram 2 shows the curve $y = x^2 + 2$ *which is intersects the straight line* $\frac{x}{2} + \frac{y}{6} = 1$

at point A.



Rajah 2

Diagram 2

(*a*) Cari koordinat titik A. [2 *markah*]

Find the coordinate of point A. [2 *marks*]

(*b*) Hitung luas rantau berlorek. [3 *markah*]

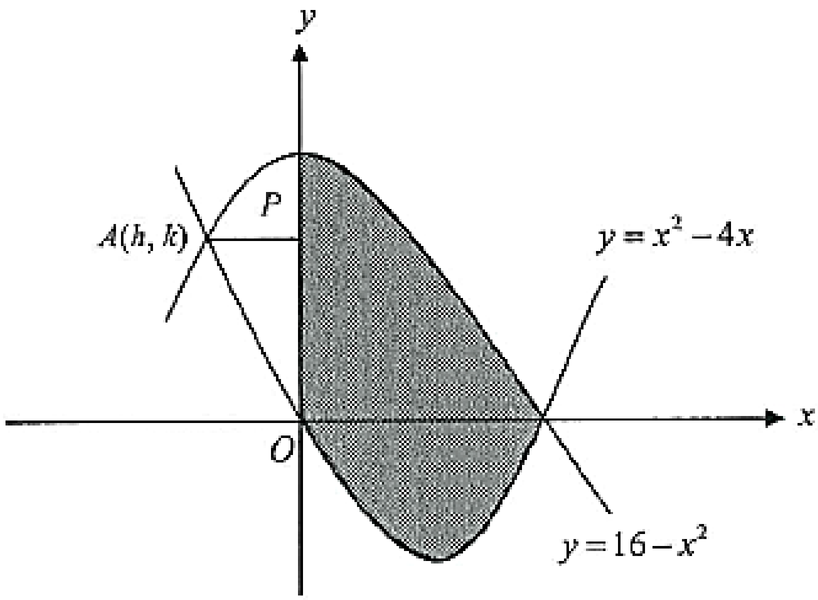
Calculate the area of shaded region. [3 *marks*]

(*c*) Diberi bahawa isipadu janaan apabila rantau yang dibatasi oleh lengkung $y = x^2 + 2$, paksi-y dan garis lurus $y = k$, dikisarkan melalui 360° pada paksi-y ialah $\frac{\pi}{8}$. Cari nilai k. [3 *markah*]

It is given that the volume generated when the region which is bounded by the curve $y = x^2 + 2$, *the y-axis and the straight line* $y = k$, *is revolved through* 360° *about the y-axis is* $\frac{\pi}{8}$. *Find the value of* k. [3 *marks*]

Rajah 6 menunjukkan lengkung $y = 16 - x^2$ dan $y = x^2 - 4x$.

Diagram 6 shows the curve $y = 16 - x^2$ and $y = x^2 - 4x$.



Rajah 6

Diagram 6

Cari

Find

(*a*) nilai h dan nilai k, [2 *markah*]

the value of h and of k, [2 *marks*]

(*b*) luas kawasan berlorek, [4 *markah*]

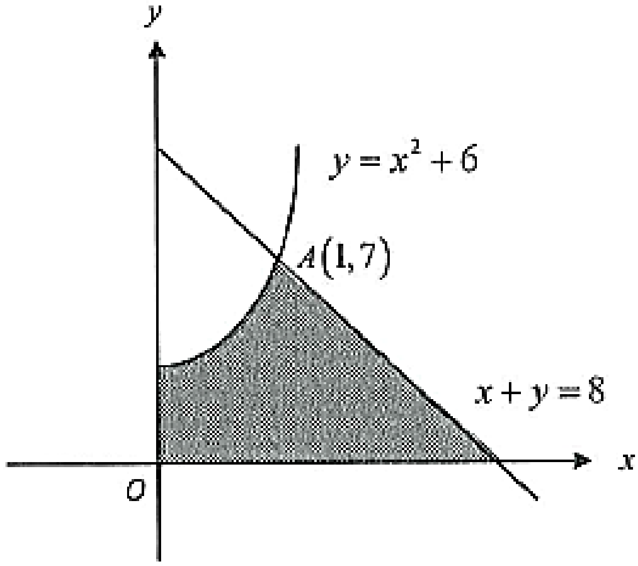
the area of the shaded region, [4 *marks*]

(*c*) isipadu yang dijanakan, dalam sebutan π, apabila rantau P dikisarkan 360° pada paksi -y. [4 *markah*]

the volume generated, in terms of π, *when the region P is revolved through* 360° *about y- axis.* [4 *marks*]

Rajah 4 menunjukkan garis lurus $x + y = 8$ menyilang lengkung $y = x^2 + 6$ pada titik A.

Diagram 4 shows the straight line $x + y = 8$ intersects the curve $y = x^2 + 6$ at point A.



Rajah 4

Diagram 4

Hitung

Calculate

(*a*) luas rantau berlorek, [4 *markah*]

the area of shaded region, [4 *marks*]

(*b*) isi padu janaan, dalam sebutan π, apabila kawasan yang dibatasi oleh lengkung, paksi-*y* dan garis lurus $y = 7$ diputarkan melalui empat sudut tepat pada paksi-*y*.

[4 *markah*]

the volume generated , in term of π, when the region bounded by the curve, y-axis and straight line $y = 7$ is rotated through four right angles about the y-axis. [4 *marks*]

YIK

$m=\frac{dy}{dx}=\frac{2}{3}(3)$

$\frac{5-0}{3-k}=-\frac{1}{2}$

$k=13$

$\frac{1}{2}(10)(5)$

$\left[\frac{x^3}{9}+4x\right]_0^3$

$\frac{1}{2}(10)(5)+\left[\frac{x^3}{9}+4x\right]_0^3$

40

$V=\pi\left[\frac{x^5}{45}+\frac{8x^3}{9}+16x\right]_0^3$

$V=\pi\left[\left(\frac{3^5}{45}+\frac{8(3)^3}{9}+16(3)\right)-0\right]$

$V=77.4\pi$

SBP

$A(0,1)$

$\frac{1}{4}\left[\frac{y^3}{3}-y\right]_1^3$ atau $\frac{1}{4}\left[\frac{y^3}{3}-y\right]_{-1}^1$

$2(3)-\int_0^2 f(x)dx$

$\left|\frac{1}{4}\left[\frac{y^3}{3}-y\right]_{-1}^1\right|+(6-m)-\frac{1}{4}\left[\frac{y^3}{3}-y\right]_1^3$

$\left|\frac{1}{4}\left(\left(\frac{1^3}{3}-1\right)-\left(\frac{(-1)^3}{3}-(-1)\right)\right)\right|+(6-m)-\left(\frac{1}{4}\left(\left(\frac{3^3}{3}-3\right)-\left(\frac{(1)^3}{3}-1\right)\right)\right)$

$\frac{14}{3}-m$

$\int_0^2\frac{1}{4}\pi(x^4+4x^2+4)dx$

$\frac{1}{4}\pi\left[\frac{x^5}{3}+\frac{4x^3}{3}+4x\right]_0^2$

$\frac{1}{4}\pi\left[\left(\frac{2^5}{3}+\frac{4(2)^3}{3}+4(2)\right)-(0)\right]$

$\frac{22}{3}\pi$ unit^3

MRSM

$\left[3x^2-x\right]_0^1$

$\left(3(1)^2-(1)\right)-\left(3(0)^2-(0)\right)$

2

$3(5)-h=\frac{7.5-0}{5-3.75}$

$7.5=\frac{3}{2}(5)^2-9(5)+c$

$y=\frac{3}{2}x^2-9x+15$

SABK S1

$7=4^2-k$

$k=9$

$(x-3)(x+3)=0$

$x=-3, x=3$

$P(3,0)$

$\left|\int_0^3 x^2-9dx\right|+\int_3^4 x^2-9dx$

$\left|\left[\frac{x^3}{3}-9x\right]_0^3\right|$ atau $\left[\frac{x^3}{3}-9x\right]_3^4$

$\left[\frac{3^3}{3}-9(3)\right]-(0)$ atau $\left[\frac{4^3}{3}-9(4)\right]-(-18)$

Luas kawasan berlorek = $18+\frac{10}{3}=\frac{64}{3}$

$\pi\int_{-9}^0 y+9\,dy$

$\pi\left[\frac{y^2}{2}+9y\right]_{-9}^0$

$\pi\left[0-\left(\frac{(-9)^2}{2}-81\right)\right]$

$\frac{81}{2}\pi$ unit^2

SABK S3

$y = \frac{x^2}{2} + 1$

$\frac{dy}{dx} = 2$

$m_2 = -\frac{1}{2}$

$\frac{-1}{2} = \frac{3-0}{2-p}$

$p = 8$

$A_1 = \int_0^2 (\frac{x^2}{2} + 1)dx$

$= \left[\left(\frac{x^3}{3} + x\right)\right]_0^2$

$= \left(\frac{2^3}{3} + 2\right) - \left(\frac{0^3}{3} + 0\right)$

$= \frac{10}{3}$

$A_2 = \frac{1}{2}(8-2)(3) = 9$

$A_1 + A_2 = \frac{37}{3}$

$v = \pi \int_1^3 2y - 2\, dy$

$= \pi[y^2 - 2y]_1^3$

$= \pi[(9-6) - (1-2)]$

$= 4\pi$

SABK S2

$y = -\frac{7}{2}x + 7;\ y = \frac{1}{2}x^2 + 3$

$-\frac{7}{2}x + 7 = \frac{1}{2}x^2 + 3$

$x = -8 \quad x = 1$

$y = 35 \quad y = \frac{7}{2}$

$P(-8, 35)\ Q\left(1, \frac{7}{2}\right)$

Luas trapezium – Luas B

$\frac{1}{2}(35 + 3.5) \times 9 - \int_{-8}^{1} \frac{1}{2}x^2 + 3dx$

$= 173.5 - \left[\frac{x^3}{3} + 3x\right]_{-8}^{1}$

$= 173.5 - \left[\left(\frac{1^3}{3} + 3(1)\right) - \left(\frac{(-8)^3}{3} + 3(-8)\right)\right]$

$= 61$

SABK TRIAL

Cari had / punca DAN pemfaktoran

Find the limits / roots AND factorisation

$f(x) = (-x-2)(x+5)$

Pengamiran mana-mana dua luas

Integration any two areas

$\left|\int_{-7}^{-5} -x^2 - 7x - 10\, dx\right| = \left|\left[-\frac{x^3}{3} - \frac{7x^2}{2} - 10x\right]_{-7}^{-5}\right|$

@

$\left|\int_{-5}^{-2} -x^2 - 7x - 10\, dx\right| = \left|\left[-\frac{x^3}{3} - \frac{7x^2}{2} - 10x\right]_{-5}^{-2}\right|$

@

$\left|\int_{-2}^{0} -x^2 - 7x - 10\, dx\right| = \left|\left[-\frac{x^3}{3} - \frac{7x^2}{2} - 10x\right]_{-2}^{0}\right|$

Ganti had ke dalam mana-mana fungsi

Substitute limit to any functions

$\left(-\frac{(-5)^3}{3} - \frac{7(-5)^2}{2} - 10(-5)\right) - \left(-\frac{(-7)^3}{3} - \frac{7(-7)^2}{2} - 10(-7)\right) = \frac{26}{3}$

@

$\left(-\frac{(-2)^3}{3} - \frac{7(-2)^2}{2} - 10(-2)\right) - \left(-\frac{(-5)^3}{3} - \frac{7(-5)^2}{2} - 10(-5)\right) = \frac{9}{2}$

@

$\left(-\frac{(0)^3}{3} - \frac{7(0)^2}{2} - 10(0)\right) - \left(-\frac{(-2)^3}{3} - \frac{7(-2)^2}{2} - 10(-2)\right) = \frac{26}{3}$

Jumlah luas / *Total area*

$= \frac{26}{3} + \frac{9}{2} + \frac{26}{3}$

$= \frac{131}{6}$

MIMS S1

$\frac{x}{2} + \frac{x^2 + 2}{6} = 1$

$A(1,3)$

$\left[\frac{x^3}{3} + 2x\right]_0^1 \quad or \quad \frac{1}{2}(1)(3)$

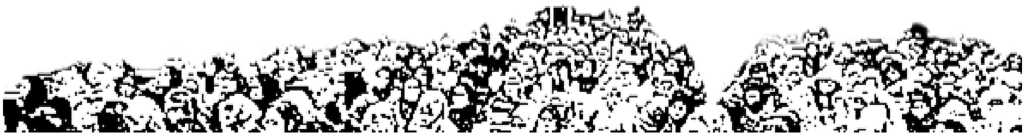
$\left[\left(\frac{(1)^3}{3} + 2(1)\right) - \left(\frac{0}{3} + 2(0)\right)\right] + \frac{3}{2}$

$\frac{23}{6}$ // 3.833

$\pi\left[\frac{y^2}{2} - 2y\right]_2^k = \frac{\pi}{8}$

$\pi\left[\left(\frac{k^2}{2} - 2k\right) - \left(\frac{2^2}{2} - 2(2)\right)\right] = \frac{\pi}{8}$

$k = 2.5$



MIMS S2

$16 - x^2 = x^2 - 4x$

$x^2 - 2x - 8 = 0$

$(x+2)(x-4) = 0$

$x = -2, x = 4$

Di titik A, $y = 16 - (-2)^2 = 12$

$h = -2, k = 12$

2

$$\int_0^4 (16x - x^2)dx + \int_0^4 (x^2 - 4x)dx$$

$$\left[16x - \frac{x^3}{3}\right]_0^4 + \left[\frac{x^3}{3} - \frac{4x^2}{2}\right]_0^4$$

$$\left[\left(16(4) - \frac{(4)^3}{3}\right) - \left(16(0) - \frac{(0)^3}{3}\right)\right] + \left[\left(\frac{(4)^3}{3} - \frac{4(4)^2}{2}\right) - \left(\frac{(0)^3}{3} - \frac{4(0)^2}{2}\right)\right]$$

$$\frac{128}{3} + \left|-\frac{32}{3}\right|$$

$$\frac{160}{3} \; // \; 53\frac{1}{3} \; // \; 53.33 \text{ unit}^2$$

4

$$Isipadu = \pi \int_{12}^{16} (16 - y)dy$$

$$= \pi\left[\left(16y - \frac{y^2}{2}\right)\right]_{12}^{16}$$

$$= \pi\left[\left(16(16) - \frac{(16)^2}{2}\right) - \left(16(12) - \frac{(12)^2}{2}\right)\right]$$

$$= 8\pi \; unit^3$$

4

MIMS S3

$$\int_0^1 (x^2 + 6)\,dx + \int_1^8 (8 - x)\,dx \;\; OR \;\; \int_0^1 (x^2 + 6)\,dx + \left[\frac{1}{2}(7)(7)\right]$$

$$\left[\frac{x^3}{3} + 6x\right]_0^1 + \left[8x - \frac{x^2}{2}\right]_1^8$$

$$\left[\frac{1^3}{3} + 6(1) - 0\right] + \left[\left(8(8) - \frac{8^2}{2}\right) - \left(8(1) - \frac{1^2}{2}\right)\right]$$

$$30\frac{5}{6} \; // \; \frac{185}{6} \; // 30.833 \text{unit}^2$$

$$\pi \int_6^7 (y - 6)\,dy$$

$$\pi\left[\frac{y^2}{2} - 6y\right]_6^7$$

$$\pi\left[\left(\frac{7^2}{2} - 6(7)\right) - \left(\frac{6^2}{2} - 6(6)\right)\right]$$

$$\frac{1}{2}\pi \text{unit}^3$$